

Numerical Integration

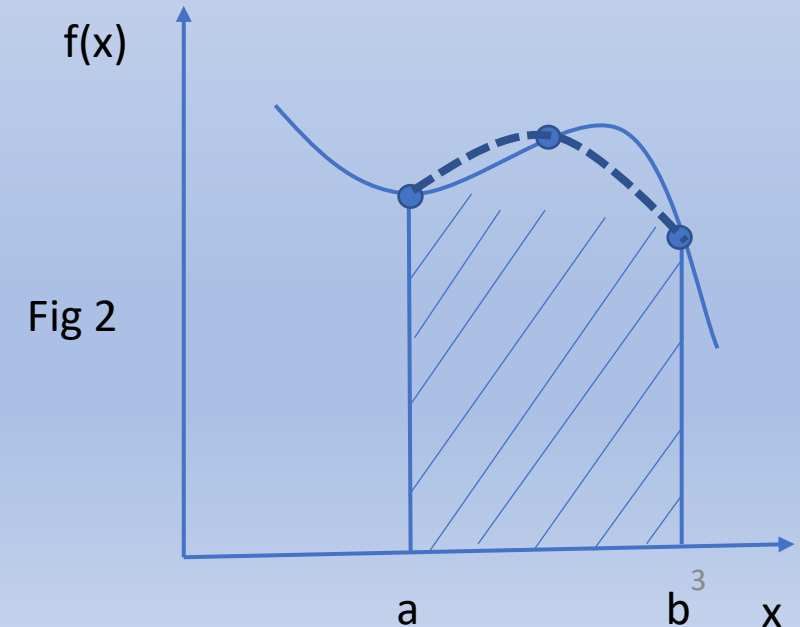
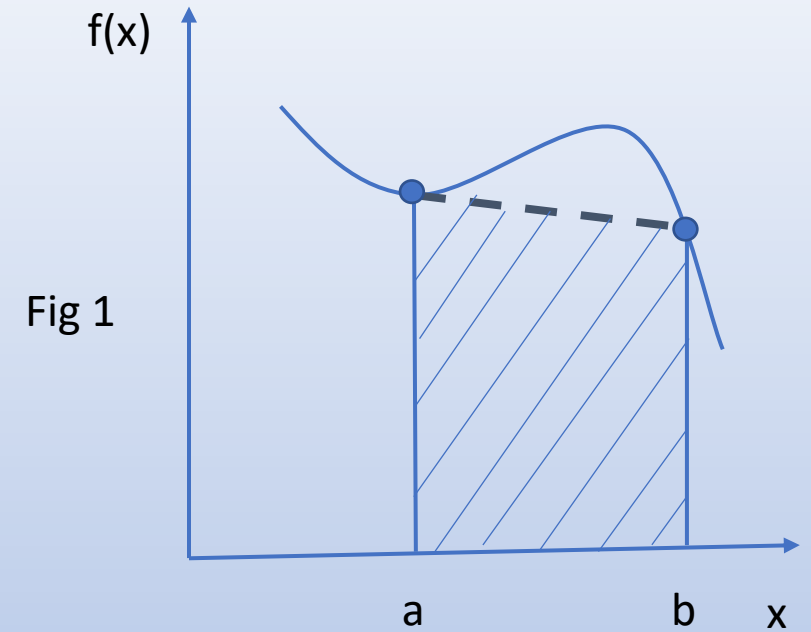
Simpson's $1/3$ Rule

Objectives

- Numerically integrate a function using Simpson's 1/3 Rule
- Simpson's 1/3 rule falls under the closed form of the Newton-Cotes integration formulas.
- Newton-Cotes formulas are the most common numerical integration schemes.
- Here we replace a complicated function or tabulated data using an approximating function that is easy to integrate.

Newton – Cotes Formula

- $I = \int_a^b f(x) dx \sim = \int_a^b f_n(x) dx$
- $f_n(x)$ – polynomial of the form
- $f_n(x) = a_0 + a_1 * x + \dots + a_{n-1} * x^{n-1} + a_n * x^n$
- n – order of the polynomial
- $n = 1$ in Fig 1 (1st order polynomial - Straight line)
- $n = 2$ in Fig 2 (2nd order polynomial - Parabola)



Simpson's 1/3 Rule

- $n = 2$ in Fig 2 (2nd order polynomial - Parabola)
- Simpson's 1/3 rule is obtained by fitting a second-degree polynomial to three equally spaced discrete points. Note that the total number of increments must be even.
- $I = \int_a^b f(x) dx = \int_a^b f_2(x) dx$
- $f_2(x)$ is represented by a second-order Lagrange polynomial
- For a single interval of two increments:
- $I = \left(\frac{h}{3}\right) * (f_0 + 4 * f_1 + f_2)$

Simpson's 1/3 Rule

- The composite Simpson's 1/3 rule for equally spaced points is obtained by applying previous equation over the entire range of integration. Note that the total number of increments must be even.
- $I = \left(\frac{h}{3}\right) * (f_0 + 4 * f_1 + 2 * f_2 + 4 * f_3 + \dots + 4 * f_{n-1} + 2 * f_n)$
- Individual error, $E_t = -\frac{(b-a)^5}{(180n^4)} * (f_m^{(4)})$;
- Global error – $O(h^4)$; h – interval size
- n – no of segments
- If n is doubled, the error term becomes one 16th of the original error value
- So to reduce the error term, the no. of segments, n need to be increased,

Simpson's 1/3 Rule

- Example – Integrate the function
- $f(x) = 0.2 + 25*x - 200 * x^2 + 675*x^3 - 900*x^4 + 400*x^5;$
- From $a = 0$ to $b = 0.8$
- The value obtained analytically is 1.640533
- Evaluate the integral by varying the no. of segments, n

Summary

In this video,

- We presented Simpson's 1/3 Rule to numerically integrate a function $f(x)$.
- The error is $O(h^4)$.
- In the next video we can look at other methods such as Simpson's 3/8 rules.